LOW-DENSITY JETS BEYOND A SONIC NOZZLE AT LARGE PRESSURE DROPS

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The gas-dynamical structure of jets of a low-density diatomic gas beyond a sonic nozzle at large pressure drops under conditions of a transition from continuous medium processes to rarefied gas processes is examined on the basis of experimental data obtained in low-density gas-dynamical tubes using electron-beam diagnostics and the Pitot tube method. Isomorphism* is shown in the density distribution and total pressure in all cross sections of the jet with respect to pressures at a constant value of the complex $R_L = R_* / N^{1/2}$ (R_* is the Reynolds number in the critical cross section of the nozzle, and N is the ratio of the Pitot pressure and the pressure in the discharge chamber). It is shown on the basis of a comparison of local Reynolds numbers for all zones of the jet that this is an analog complex. The experimental data on the variation in the jet structure are presented as a function of the number R_L in the range of 5-600. For $R_L > 100$ the flow in the jet can be considered as continuous; for $R_L < 5$-10 the flow corresponds to a scattering process; the range of $5\text{-}10 < R_L < 100$ corresponds to a transitional state. Ranges of isomorphism of the jet with respect to R_* and N are indicated. Based on the results of the measurements, it is shown that the flow behind a Mach disk for $R_L > 200$ remains subsonic on the axis to a distance of several lengths of the primary cycle. A transition to supersonic velocity on the jet axis can occur with a decrease in the numbers R_L owing to ejection acceleration by the supersonic ring-shaped compressed layer.

1. A description of the equipment and procedure of the experiments is contained in [1-3]. Sonic nozzles of the type of an opening in a thin wall were used in the experiments, with a ratio of wall thickness to opening diameter of less than 0.05. For the Reynolds numbers R_*, calculated from the parameters at the mouth of the nozzle, the condition $R_* \geqslant 200$ was satisfied; in this case one can neglect the boundary layer effect at the nozzle and take the flow rate coefficient of the nozzle as equal to unity [4]. Nitrogen, air, and carbon dioxide were used as the working gas at a total temperature of $T_0 \approx 300°K$.

It is known from calculations of a jet of nonviscous gas and from experimental studies (see [4-6], for example) that for large ratios of the total pressure p_0 to the pressure p_j in the space being flooded the characteristic geometrical dimensions L_i of different zones of the initial part of the jet (the diameter of the suspended pressure drop, the distance to the Mach disk, the thickness of the compressed layer behind the suspended pressure drop, etc.) obey the functions $L_i \sim d_* N^{1/2}$, where d_* is the diameter of the critical cross section of the nozzle and $N = p_0/p_j$. The given relationship is satisfied more exactly for larger N. Thus, according to calculations ignoring viscosity [7] for a fixed Mach number M at the nozzle mouth and a fixed ratio γ of specific heat capacities, the flow in the flooding jet in the coordinates $\xi = x/d_* N^{1/2}$, $\eta = y/d_* N^{1/2}$ asymptotically approaches isomorphism with an increase in N. In this case the gas-dynamical parameters also tend toward isomorphism, except for the low-entropy layer near the boundary of the jet. This is connected with the formation in the zone bounded by the suspended pressure drop of flow from an extended source in which the velocity of the escaping gas reaches its limiting value along the flow lines.

For a jet of real gas at low densities, the given flow pattern is complicated by viscosity effects and relaxation processes connected with internal degrees of freedom. Besides the pressure ratio N, the Mach

*This word is apparently interchangeable with "self-similarity" – Translator.

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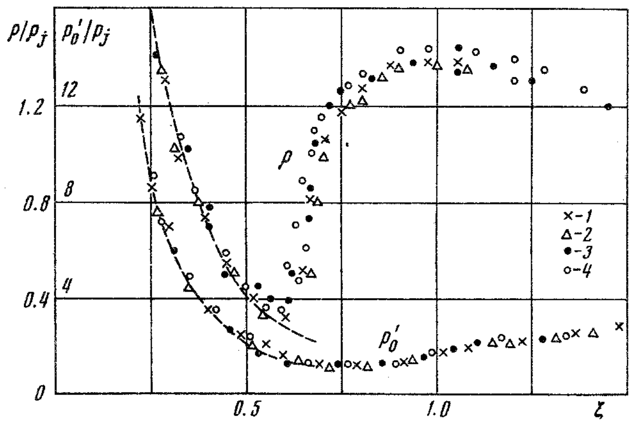

Fig. 1

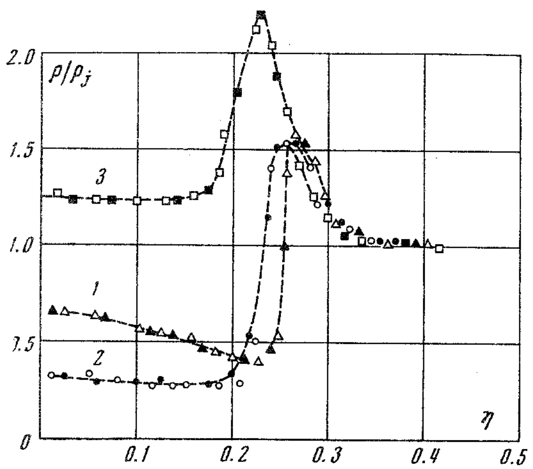

Fig. 2

number M, and the heat capacity ratio γ, the flow depends on the Reynolds, Prandtl, and Schmidt numbers and the temperature factor T_0/T_j.

An analysis of various literature data and the experiments of the present report showed that in the transition to rarefied gas states the nature of the flow in the different zones of the initial section of the jet depends essentially on the value of the local Reynolds number R_i, which is constructed from the values of the density ρ_i, velocity u_i, coefficient of viscosity μ_i, and dimension L_i characteristic for each zone. Let us examine this criterion in turn for the zone of the jet core bounded by the suspended pressure drop and the Mach disk (zone 1), the zone of the mixing layer behind the suspended pressure drop (zone 2), and the zone behind the Mach disk (zone 3). We will take the law of variation of the coefficient of viscosity with temperature in the form $\mu \sim T^{\omega}$.

The following dependence [8] can occur for the density in zone 1 near the axis of the jet at a large distance from the nozzle mouth where the velocity u_1 is close to the maximum velocity:

$$\rho/\rho_0 = Bd_*^2/x^2 \tag{1.1}$$

Here ρ_0 is the density in the forechamber of the nozzle, and $B = B(\gamma, M)$. We will take the distance from the nozzle mouth to the Mach disk as the characteristic dimension of this zone. According to [4]

$$L_1 = 0.67\ d_*\ N^{1/2} \tag{1.2}$$

and for the Reynolds number constructed from the local values of ρ_1, u_1, μ_1, and the length L_1 we find

$$R_1 \sim M_1^{2\omega} R_* / N^{1/2} \tag{1.3}$$

where M_1 is the Mach number at the point x. We note that this Reynolds number characterizes the longitudinal dissipative effects in one-dimensional flow. On the other hand, we also introduce the local Knudsen number $K_M = l_M/x$, determined from the mean free path of the molecules in the direction of flow of the gas $l_M = lM$ [9] (l is the mean free path in Lagrangian coordinates). Since

$$R_1 \sim M_1^2/K_M \tag{1.4}$$

while $\omega \approx 1$ at low values of the static temperature, from a comparison of (1.3) and (1.4) we have

$$K_M \sim N^{1/2}/R_*$$

The complex $R_*/N^{1/2}$ is convenient for use in describing the effects of viscosity and thermal conduction [10, 11], as well as relaxation effects in the core of the jet. Thus, according to the estimate suggested in [12], the "freezing in" of rotational and translational degrees of freedom of the molecules occurs at a distance from the nozzle mouth determined by the relation

$$(x_f/d_*)^{\omega+(2\gamma-1)(\omega-1)} \sim (ZK_*)^{-1} \tag{1.5}$$

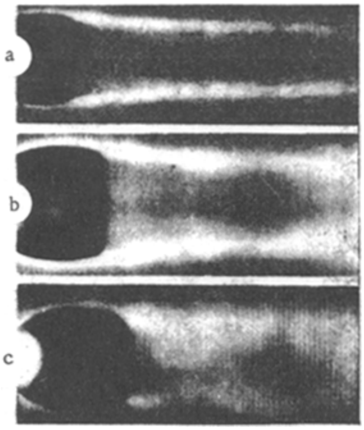

Fig. 3

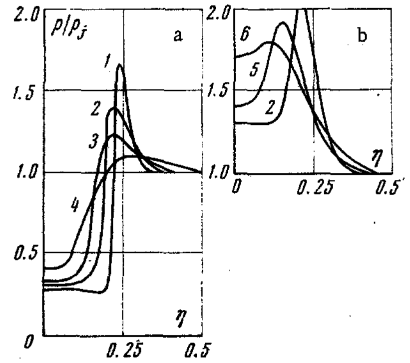

Fig. 4

in which $K_* = l_*/d_*$ is the Knudsen number with respect to conditions in the critical section, and Z is the number of collisions necessary to establish equilibrium with respect to rotational or translational degrees of freedom. At $\omega = 1$,

$$\xi_f = x_f/d_* N^{1/2} \sim R_*/ZN^{1/2} \quad (1.6)$$

i.e., at an arbitrary N but a fixed value of the complex $R_*/N^{1/2}$, the freezing in occurs in congruent cross sections $\xi_f = \text{const}$.

In zone 2 the Reynolds number can be determined from the distance from the nozzle along the boundary of the jet $R_{2S} = \rho_2 u_2 s/\mu_2$, where ρ_2, u_2, and μ_2 are some average values of the density, velocity, and coefficient of viscosity in the mixing layer in the given cross section. At a large enough distance from the nozzle, $s \approx x$. Having made use of the condition according to which at $x/d_* \gg 1$ the flow rate through the compressed layer comprises the greater share of the total flow rate, we find the average value $\rho_2 u_2$ from the continuity equation

$$\rho_2 u_2 \sim \rho_* u_* d_*^2 / D\Delta \quad (1.7)$$

where D and Δ are the diameter of the suspended pressure drop and the thickness of the compressed layer behind the suspended pressure drop, respectively. Considering that $D \sim d_* N^{1/2}$, $\Delta \sim d_* N^{1/2}$, we obtain

$$R_{2x} \sim \mu_0 R_* / \mu_2 N^{1/2} \quad (1.8)$$

For μ_2 one can take the value corresponding to the average temperature in the mixing layer, equal, for example, to the arithmetic mean of the temperature T_s beyond the suspended pressure drop and the temperature T_j in the submersion space. At large enough values of N and x/d_*, the condition $T_s \ll T_j$ often occurs. Then

$$R_{2x} \sim (T_0/T_j)^\omega R_*/N^{1/2} \quad (1.9)$$

At $T_0 = T_j$ in the congruent cross sections $R_{2x} \sim R_*/N^{1/2}$.

In zone 3 we taken the diameter D_M of the Mach disk as the characteristic dimension for the Reynolds number $R_3 = \rho_3 u_3 D_M/\mu_3$. Here, when $N \gg 1$, the following conditions occur:

$$\mu_3 \sim \mu_*, \quad u_3 \approx u_*/u_{\max}, \quad \rho_3 \approx (\gamma + 1) B\rho_0 d_*^2/(\gamma - 1) L^2$$

Using Eq. (1.2) and the condition $D_M \sim d_* N^{1/2}$, we obtain

$$R_3 \sim R_*/N^{1/2} \quad (1.10)$$

The variation in the ratio of the thickness of the shock waves to the characteristic dimension of the jet is described in [3, 13] by the dimensionless numbers $K_0 N^{1/2}$ and $T_0/d_* (p_0 p_j)^{1/2}$ ($K_0 = l_0/d_*$ is the Knudsen number and l_0 is the mean free path in the stagnation chamber). It is easy to show that at $T_0 = T_j$ these complexes are inversely proportional to $R_*/N^{1/2}$.

Thus, the local Reynolds numbers for the zones being considered and for the different physical processes prove to be proportional to $R_*/N^{1/2}$. We shall take this complex as the determining criterion and will henceforth designate it as R_L.

According to the results of [14], the flow in the mixing layer near the boundary of the jet is laminar for $T_0 = T_j$ and $R_L \lesssim 10^3$, and in the range of $50 < R_L < 10^3$ at fixed values of R_L an isomorphic variation occurs in all the geometrical dimensions of the initial section of the jet with respect to $N = p_0/p_j$, when $N \geq 100$.

The experiments of the present report pursued the following objectives:

1) to study the structure of the longitudinal and transverse distributions of the gas-dynamical parameters in the initial section of the jet and in particular to determine whether these distributions obey an isomorphous law of variation as a function of p_0/p_j at $R_L = \text{const}$;

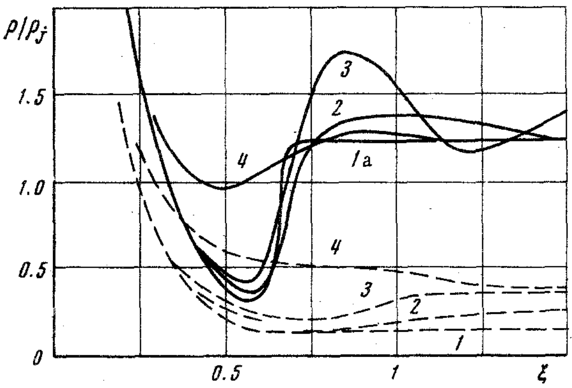

Fig. 5

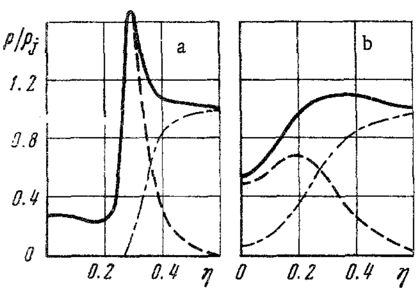

Fig. 6

2) to investigate in detail the reorganization of the flow pattern in the jet upon a decrease in R_L from 10^3 to values corresponding to scattering processes [13], during which the local free path length of the molecules becomes comparable with the characteristic dimensions of the flow. Some new information on the nature of the flow in the region of the Mach disk was obtained in the realization of this program.

2. In order to test the isomorphism with respect to N of the distributions of gas-dynamical parameters in the initial section of the jet in the laminar flow state, series of experiments were conducted in each of which for the case $T_0 = T_j$ the number R_L was kept constant during the variation of the numbers R_* and N in a broad range ($R_* > 100$, $N > 100$). It was established from the data of the measurements that in generalized coordinates the longitudinal and transverse fields of density ρ and total head pressure p_0' actually are isomorphous with respect to N at a fixed R_L. In the analysis coordinates were chosen such that the axial distributions of ρ and p_0' in the isentropic core had a universal form. According to Eq. (1.1), for the axial density distribution

$$\frac{1}{\xi^2} = \frac{\rho}{\rho_0}\frac{N}{B} = \frac{\rho}{\rho_j}\frac{T_0}{T_j}\frac{1}{B} \tag{2.1}$$

For the case of large numbers M in front of the Pitot tube $p_0' \approx \rho u_m^2$, from which, in light of the foregoing,

$$\frac{1}{\xi^2} = \frac{\gamma - 1}{2\gamma B}\frac{p_0'}{p_j} \tag{2.2}$$

Thus, the experimentally measured distributions of ρ and p_0' constructed in the form of dependences of $\rho N(\rho_0 B)^{-1}$ and $p_0'(Bp_j)^{-1}$ on the coordinates ξ and η are clearly generalized to the hypersonic region. For fixed values of M, γ, and T_0/T_j, for purposes of simplicity and clearness it is sufficient to construct dependences of the ratios ρ/ρ_j and p_0'/p_j on these same coordinates ξ and η. The results of measurements of the longitudinal axial distributions of density and total pressure in a nitrogen jet at $R_L = 140$ are presented in Fig. 1. The designations in this figure correspond to the following experimental parameters N and R_*, respectively: 1) 5000 and 10,300; 2) 900 and 4100; 3) 360 and 2460; 4) 100 and 1460. The dashed curves correspond to the axial distributions of ρ and p_0' corresponding to the escape of a nonviscous diatomic gas into a vacuum.

The results of the measurement of transverse density distributions in a nitrogen jet at $R_L = 110$ in three different cross sections: 1) $\xi = 0.38$, 2) $\xi = 0.55$, 3) $\xi = 0.75$ are shown as an example in Fig. 2. Here the light points correspond to $N = 13{,}500$ and $R_* = 1260$ while the dark points correspond to $N = 3040$ and $R_* = 6030$. Analogous designations are used for other values of R_L. On the basis of the experimental data obtained, it can be asserted that in the case of a laminar state of flow in the initial section of the jet at fixed values of T_0/T_j and R_L, the density distribution is isomorphous with respect to N if $N \geq 100$.

It should be mentioned that this conclusion is qualitatively confirmed by series of experiments on various instruments differing in the configuration of the walls of the pressure chamber, the location of the nozzles, the measuring apparatus, and the flow rate characteristics (from 0.1 to 1 g/sec). However, it was

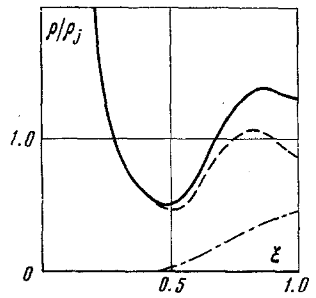

Fig. 7

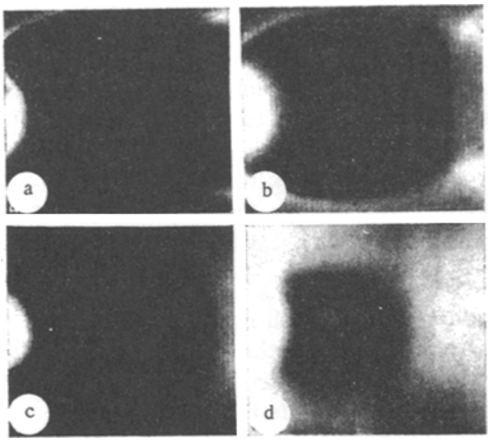

Fig. 8

also found that among some of the results obtained on different instruments small differences can occur in the characteristic geometrical dimensions of the jet (up to 20%) with corresponding differences in the distributions of gas-dynamical parameters. Apparently, one of the reasons for this is connected with the appearance in the space surrounding the jet of weak ejection fluxes differing in nature and intensity which could act directly on the jet itself and lead to errors in determining the true values of the pressure p_j and, accordingly, N, ξ, and η.

3. A considerable number of experiments were conducted for the purpose of studying the reorganization of the flow pattern upon a decrease in the number R_L. Their analysis is greatly facilitated thanks to the isomorphism described above.

As an example, schlieren photographs of the flow in a nitrogen jet $R_L = 365$, 280, and 130 are presented in Fig. 3. The rapid changes in compression of the "barrel-shaped" structure of the initial section, the boundary of the jet, and the extended annular layer of dense gas located behind the Mach disk are well seen. The flow in this annular layer is rather viscous, which is qualitatively confirmed by the increase in its thickness with a decrease in R_L which is evident to the eye.

The results of quantitative studies of the transition of flow in the jet from the laminar flow state of a continuous medium to the rarefied gas states are presented in Figs. 4 and 5. The measured transverse density distributions in two cross sections, a) $\xi = 0.6$ in front of the Mach disk and b) $\xi = 0.83$ behind the Mach disk, are given in Fig. 4, a and b, respectively. The curves in these figures correspond to the following numbers R_L: 1) 440, 2) 140, 3) 70, 4) 30, 5) 85, and 6) 40.

Figure 5 demonstrates the change in the longitudinal axial distributions of ρ (solid curves) and p_0' (dashed curves) upon a decrease in R_L from 600 to 10. Here curves 1, 1a, 2, 3, and 4 correspond to R_L values of 580, 365, 140, 65, and 13, respectively. Each of the curves 1-4 for density was obtained through the averaging of isomorphous distributions obtained in several experiments at different N but at a fixed R_L, analogous to that shown in Fig. 1. A disturbance in the isomorphism with respect to N is observed in the distribution of the parameters at small R_L.

Because of the effect of corrections to the Pitot tube readings, the disturbance in isomorphism in the distribution of measured p_0' at small R_L sets in at larger R_L than for the density distribution. In view of this, the curves of p_0' distributions in Fig. 5 are given only for N = 5000.

The results of the measurement of the density and total pressure show that for $R_L \geq 200$ in the zone bounded by the viscous annular layer at a large distance behind the Mach disk (the measurements were conducted up to $\xi = 2.5$), the alternating rises and falls in ρ and p_0' are not so great that one could speak of a transition through the speed of sound. This leads to the conclusion that a rather extended (on the order of several L) region of subsonic flow may exist here. The static pressure in this zone near the axis of the jet determined from density measurements is about 1.3-1.5 times greater than the pressure in the submersion space. Annular flow behind the Mach disk without a cyclic structure was also recorded in the experiments of [17]. The present results indicate that a number of the hypotheses [7, 15] relative to the nature of the flow behind the Mach disk which are adopted for the determination of its location are apparently questionable.

It follows from the results presented in Figs. 4 and 5 that in the section between the nozzle exit and the Mach disk when $R_L > 100$, the effect of viscosity is manifested for the most part only in the mixing layer beyond the suspended drop in compression. In this case, the diameter of the suspended pressure drop, the distance to the Mach disk, and the distribution of the parameters in the isentropic core for $R_L \gtrsim 200$ are close to the values calculated from a model of an ideal fluid [7], while the thicknesses of the rapid changes in compression are negligibly small.

A comparison of the measured values of the maximum density ρ_{max} in the compressed layer at $\xi = 0.6$ and $R_L = 440$ and 140 (see curves 1 and 2 in Fig. 4a) with the calculated values ρ_s behind the thin hanging pressure drop showed that $\rho_{max} > \rho_s$. At the same time, according to the experimental results presented below, the position of the inner boundary of the mixing layer proves to correspond approximately to the value $\rho \approx \rho_{max}$. On this basis, it can be assumed that for $R_L > 100$ the suspended pressure drop and the mixing layer are separated by a zone of nonviscous flow.

In order to determine the position of the inner boundary of the mixing layer, measurements were made of the partial densities of components in a nitrogen jet, containing a mixture of N_2 and CO, escaping from a sonic nozzle into the submersion space. The carbon monoxide was supplied to the pressure chamber through a special valve. The flow rates of N_2 and CO were determined from the equilibrium parameters in the critical cross section and their diameters. The results of measurements of the transverse and longitudinal distributions are presented in Figs. 6 and 7. Here the solid curves designate the total density of the mixture, the dashed curves the nitrogen density, and the dot–dash curves the CO density. The transverse distributions in Fig. 6a and b were measured in the cross section $\xi = 0.55$ and correspond to two different states: a) $R_L = 157$, $N = 8.85 \cdot 10^4$; b) $R_L = 20$, $N = 8.5 \cdot 10^4$. The parameters for the axial distribution (Fig. 7) are $R_L = 20$ and $N = 8.5 \cdot 10^4$.

It is clearly seen from Fig. 6a that for the case of $R_L = 157$ the penetration of the CO component becomes negligibly small at values of the coordinate η corresponding to $\rho = \rho_{max}$ on the total density curve, which allows one to determine this coordinate and the position of the inner boundary of the mixing layer.

The rearrangement of the described flow pattern upon a decrease in the number R_L can be traced through the series of curves in Figs. 4 and 5. Here one can see the gradual thickening of the suspended pressure drop, the Mach disk, and the mixing layer. In this case, the diameter of the suspended pressure drop decreases, while the location of the Mach disk remains practically unchanged. For $R_L < 100$, the maximum density in the compressed layer measured in the experiments becomes lower than the calculated density behind the thin transverse pressure drop, which indicates the merging of the zones of the pressure drop and the mixing layer. The compressed layer becomes completely viscous, while its thickening upon a decrease in R_L along with the thickening of the Mach disk leads to the contraction of the transverse and longitudinal dimensions of the core of the jet. A considerable change occurs simultaneously in the nature of the flow behind the Mach disk. It is seen from Fig. 4b that the thickening of the annular viscous layer results in the flow behind the Mach disk becoming completely viscous. The closing in of the mixing layer on the axis leads to an increase in the measured density and total pressure and, as follows from the longitudinal distributions of ρ and p_0' presented in Fig. 5, this increase propagates in the direction opposite to the flow upon a decrease in R_L, although it does not reach the zone of the Mach disk. At $R_L \approx 100$, the closing in of the viscous layer on the axis even occurs immediately behind the Mach disk, which at $R_L \approx 100$ cannot be considered as an isolated direct shock wave. The thickening of the viscous layer also leads to an increase in its ejecting effect on the flow near the axis, as a consequence of which a region of supersonic velocity can develop in the zone behind the Mach disk.

Upon a decrease in the number R_L to values on the order of 5-10, when the shock waves degenerate in density and the flow in the jet becomes almost completely viscous, a transition occurs to the so-called scattering process [13]. Here the mean free path of the molecules in the submersion space becomes comparable with the transverse dimensions of the compressed layer, so that the penetration of the molecules into the core of the jet begins to take on a free-molecular nature. This effect is shown in Fig. 6b and Fig. 7 for $R_L = 20$.

The described reorganization of the flow pattern upon a transition to small values of R_L is illustrated qualitatively in Fig. 8 by a series of schlieren photographs of the jet flow at $N \approx 3000$ and $R_L = 155$, 77, 40, and 20.

Flow into vacuum occurs in the limiting case $R_L \to 0$.

4. It was noted above that a departure from isomorphism with respect to N was observed in the distribution of the parameters at low values of the number R_L. The results obtained in the work make it possible to show the reasons for these disturbances.

We note first of all that the isomorphism of the distributions of the parameters with respect to N in the initial section has an approximate character, being more fully realized for larger N and greater distances of the given cross section from the nozzle exit. At small distances from the exit, where the velocity of the escaping gas is still appreciably less than the limiting velocity, and the density distribution function differs from the function (1.1) corresponding to flow from a fictitious source, considerable deviations from isomorphism occur.

As shown above, a decrease in the number R_L leads to contraction of the core of the jet due to the thickening of the viscous compressed layer and the Mach disk. With a decrease in R_L, it is possible for conditions to set in such that the thickening of the viscous zone disturbs the nonviscous flow in the core to the point where hypersonic flow will form in it.

An approximate estimate can be given for the boundary of the disturbance in isomorphism. For this purpose we suggest that the disturbance in flow in the core be connected with thickening of the Mach disk. Taking into account the thickening of the viscous compressed layer hardly changes the estimate which is finally obtained.

It follows from the experimental results of the present work, as well as those from a study of the spherical expansion of a gas into the surrounding space [16], that the thicknesses of the Mach disk and the spherical shock wave are described well by the expression

$$\tau/L \approx 20/(20 + R_L) \quad (4.1)$$

Assuming that the spreading of the shock wave occurs mainly upstream from the continuous position, we obtain the following estimate for the dimension x_1 of the undisturbed flow in the core:

$$x_1/d_* \approx 0.67\, N^{1/2}/(1 + 20/R_L) \quad (4.2)$$

As a measure of appreciable deviation from isomorphism we can take a 10% difference between the velocity in front of the shock wave and the maximum attainable velocity according to the stagnation parameters. Then it can be assumed with an accuracy of 10% that isomorphism occurs when $x_1/d_* > 5$. Comparing this with the latter expression, we find that corresponding to each value R_L there is a range of N in which isomorphism of the distribution of parameters with respect to N must be satisfied:

$$N > 100\,(1 + 20/R_L)^2 \quad (4.3)$$

It follows from the expression (4.3) obtained that with a decrease in the number R_L, a departure from isomorphism occurs for all higher values of N. This conclusion is confirmed by the experimental data presented above.

LITERATURE CITED

1. A. K. Rebrov, S. F. Chekmarev, and R. G. Sharafutdinov, "Effect of rarefication on structure of a free nitrogen jet," Zh. Prikl. Mekhan. i Tekhn. Fiz., No. 1 (1971).
2. V. A. Sukhnev, "Determination of corrections to Pitot tube readings in a supersonic flow of rarefied gas," Izv. Akad. Nauk SSSR, Mekhan. i Mashinostr. No. 5 (1964).
3. V. V. Volchkov and A. V. Ivanov, "Thickness and internal structure of normal drop in density which forms during the escape of a strongly underexpanded jet into a space with low density," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaz, No. 3 (1969).
4. H. Z. Ashkenas and F. S. Sherman, "The structure and utilization of supersonic free jets in low density wind tunnels," in: Rarefied Gas Dynamics, Vol. 2, Academic Press (1966).
5. S. Crist, R. M. Sherman, and D. R. Glass, "Study of the highly underexpanded sonic jet," AIAA Journal, 4, No. 1 (1964).
6. V. S. Avduevskii, A. V. Ivanov, I. M. Karpman, V. D. Traskovskii, and M. Ya. Yudelovich, "Flow in a supersonic viscous underexpanded jet," Izv. Akad. Nauk SSSR, Mekhan. Zhid. i Gaz, No. 3 (1970).
7. G. I. Averenkova, É. A. Ashratov, T. G. Volkonskaya, Yu. N. D'Yakonov, M. I. Egorova, D. A. Mel'nikov, G. S. Roslyakov, and V. I. Uskov, Supersonic Jets of Ideal Gas [in Russian], Izd-vo MGU, Moscow (1970), Part 1, p. 2.
8. M. Sibulkin and W. Callaher, "Far-field approximation for a nozzle exhausting into a vacuum," AIAA Journal, 1, No. 6 (1963).
9. M. N. Kogan, Rarefield Gas Dynamics [in Russian], Nauka, Moscow (1967).
10. V. N. Gusev and A. V. Zhbakova, "The flow of a viscous heat-conducting compressible fluid into a constant pressure medium," in: Rarefield Gas Dynamics, Vol. 1, Academic Press (1969).
11. A. K. Rebrov and S. F. Chekmarev, "Spherical escape of a viscous heat-conducting gas into a submersion space," Zh. Prikl. Mekhan. i Tekhn. Fiz., No. 3 (1971).
12. E. L. Knuth, "Supersonic molecular beams," Appl. Mech. Rev., 17, No. 10 (1964).

13. E. P. Muntz, B. B. Hamel, and B. L. Maguire, "Some characteristics of exhaust plume rarefaction," AIAA Journal, 8, No. 9 (1970).
14. V. S. Avduevskii, A. V. Ivanov, I. M. Karpman, V. D. Trashovskii, and M. Ya. Yudelovich, "Effect of viscosity on flow in the initial section of a strongly underexpanded jet," Dokl. Akad. Nauk SSSR, 197, No. 1 (1971).
15. T. C. Adamson, Jr. and J. A. Nicholls, "On the structure of jets from highly underexpanded nozzles into still air," J. Aero/Space Sci., 26, No. 1 (1959).
16. A. A. Bochkarev, A. K. Rebrov, and S. F. Chekmarev, "Hypersonic spherical expansion of a gas with a stationary shock wave," Zh. Prikl. Mekhan. i Tekhn. Fiz., No. 5 (1969).
17. G. Dupeyrat and M. Giat, "Etude de jet en atmosphère raréficé visualisation et mesura de densité," Compt. Rend. Acad. Sci., 270, No. 24 (1970).